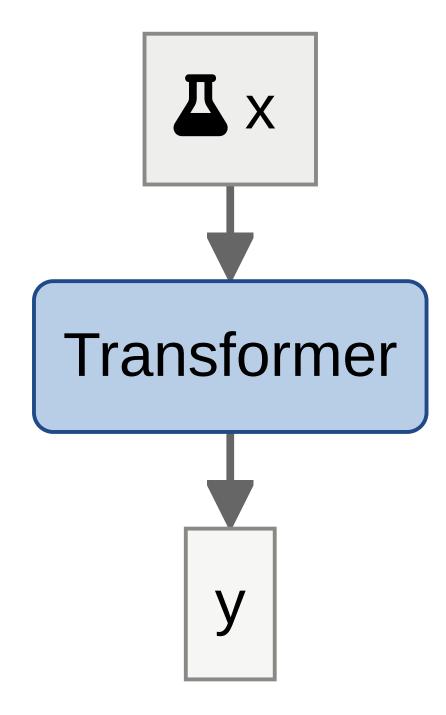
Generative Models

Recap

Model



Data



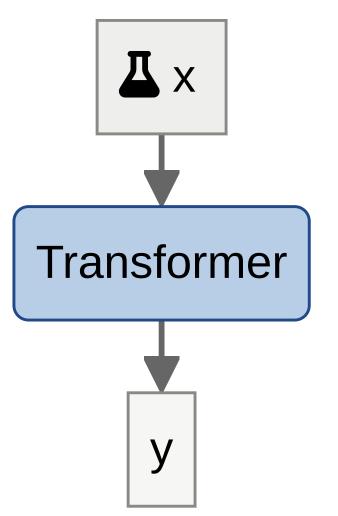
Optimization

Recap: How to train a network?

Collect Data



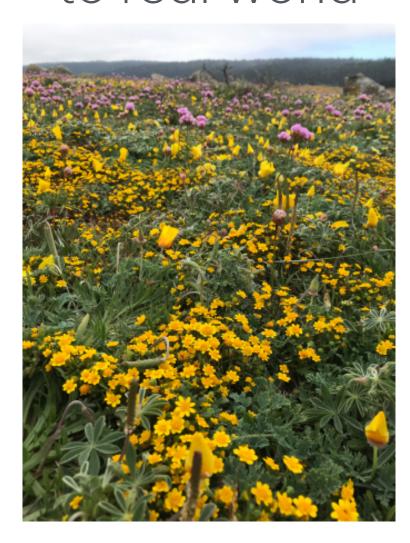
Design / download architecture



Train model



Apply model to real world



This never works !!!

Recap: How to train a network?

Training is an iterative process Step 2: Training 5-10% of work Train Step 1: Data curation Design / download model 70-80% of work architecture Дх Collect Data Transformer Apply model Look at to real world your data Step 3: Testing 15-20% of work

Part I: Done

Data

 x_i, y_i

• • •



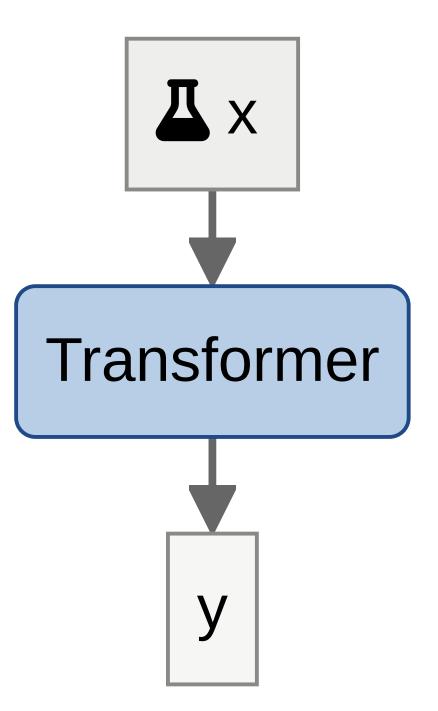
pink primrose



tiger lily

Train model

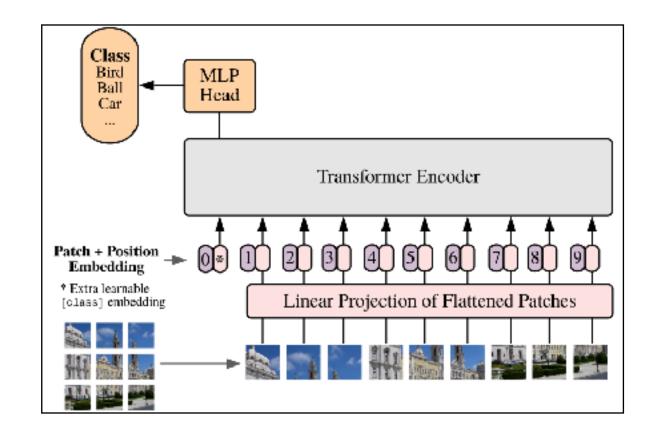
$$f: x \to y$$



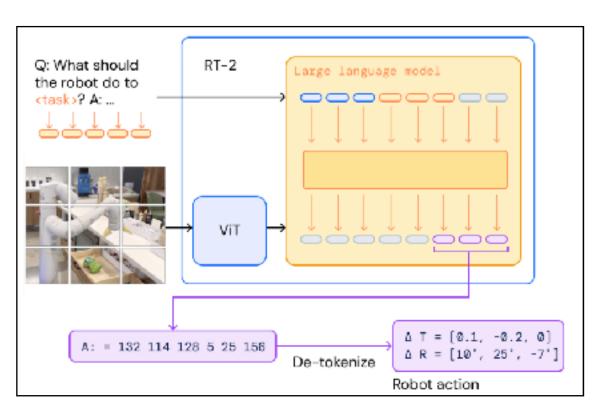
Generative Models

Discriminative models

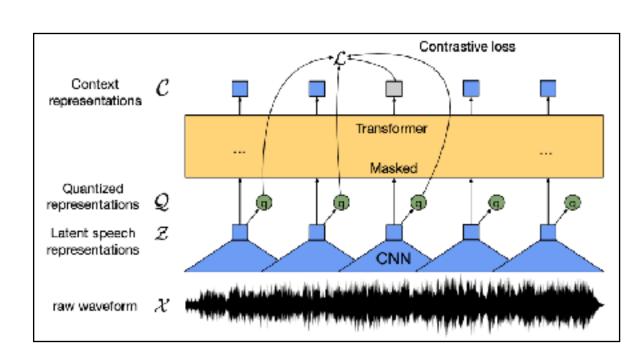
- Discriminative model: P(Y|X)
- Examples:
 - Image/video recognition
 - Speech recognition
 - Control policies
 - Weather prediction



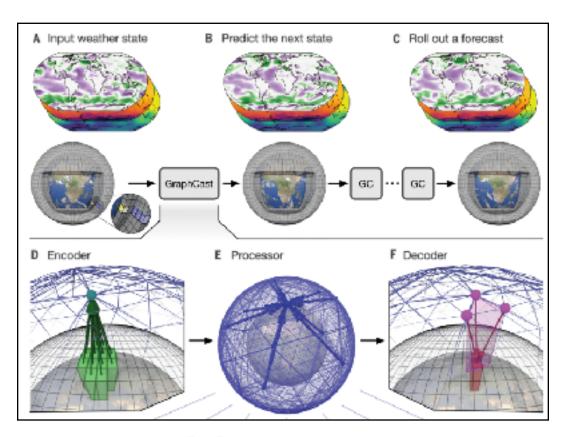
[1] Vision Transformer



[3] RT-2



[2] Wave2vec 2.0



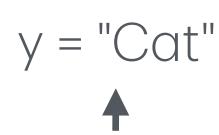
[4] GraphCast

•

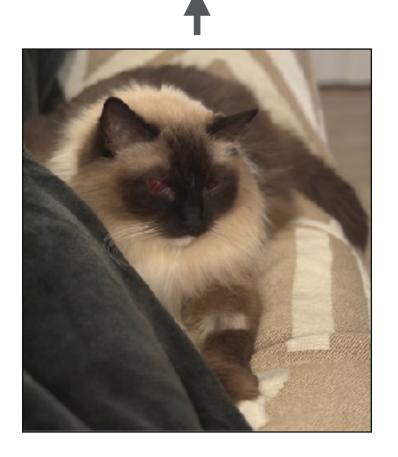
- [1] Dosovitskiy, Alexey, et al. "An Image is Worth 16x16 Words: Transformers for Image Recognition at Scale." International Conference on Learning Representations. 2020.
- [2] Baevski, Alexei, et al. "wav2vec 2.0: A framework for self-supervised learning of speech representations." Advances in neural information processing systems 33 (2020): 12449-12460.
- [3] Brohan, Anthony, et al. "Rt-2: Vision-language-action models transfer web knowledge to robotic control." arXiv preprint arXiv:2307.15818 (2023).
- [4] Remi Lam et al., Learning skillful medium-range global weather forecasting. Science 382, 1416-1421 (2023).

Discriminative models in deep learning

- Discriminative model: P(Y|X)
- Examples:
 - Image/video recognition
 - Speech recognition
 - Control policies
 - Weather prediction



Deep Network



Training







"Cat"







"Dog"







"Horse"

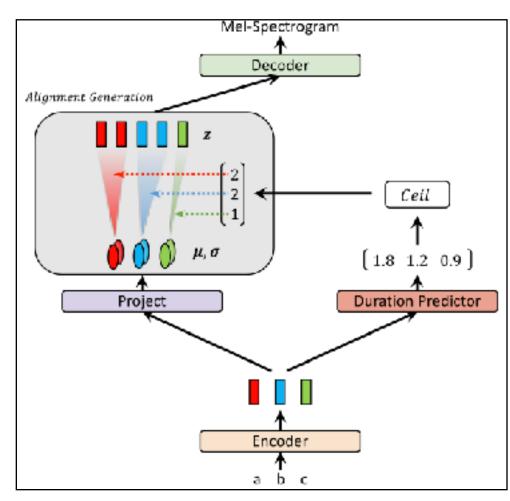
Generative models

- Generative model: P(X)
- Examples:
 - Image/video generation
 - Speech synthesis
 - Physics simulation / world modeling
 - Weather simulation (gaming)

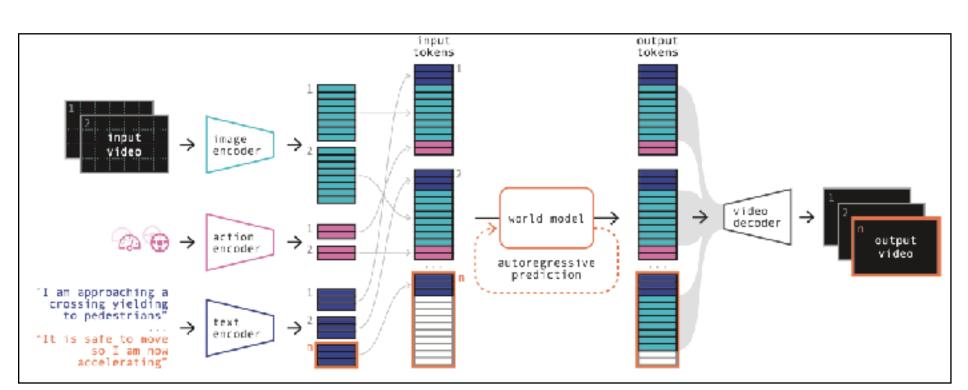




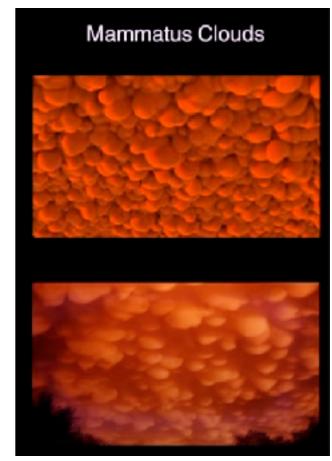
[1] Sora



[2] Glow-TTS



[3] GAIA-1



[4] Weatherscapes

^[1] Brook, Tim, et al. "Video generation models as world simulators" OpenAI Blog (2024)

^[2] Kim, Jaehyeon, et al. "Glow-tts: A generative flow for text-to-speech via monotonic alignment search." Advances in Neural Information Processing Systems 33 (2020): 8067-8077...

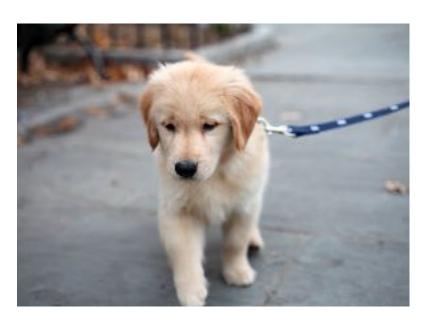
^[3] Hu, Anthony, et al. "Gaia-1: A generative world model for autonomous driving." arXiv preprint arXiv:2309.17080 (2023).

^[4]J. A. Amador Herrera, et al. "Weatherscapes: Nowcasting Heat Transfer and Water Continuity." ACM Transactions on Graphics (SIGGRAPH Asia 2021), Vol. 40, No. 6, Article 204...

Generative modeling in deep learning

- Generative model: P(X)
- Examples:
 - Image/video generation
 - Speech synthesis
 - Physics simulation / world modeling
 - Weather simulation (gaming)

•



Deep Network

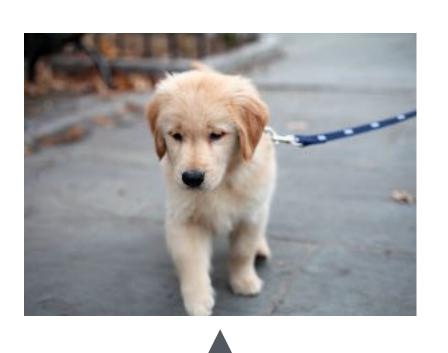


Training

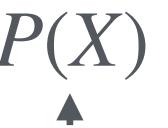


Generative models

- Two tasks of a generative model P(X)
 - Sampling: $x \sim P(X)$
 - Density estimation: P(X = x)



Deep Network

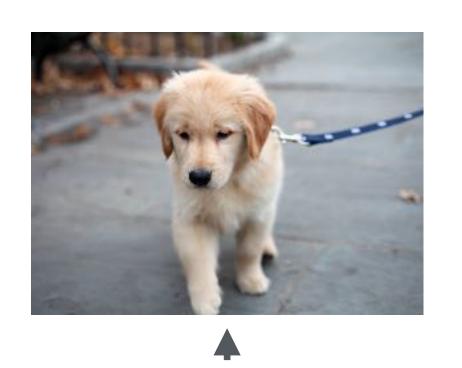


Deep Network

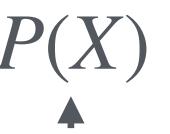


Generative modeling is hard

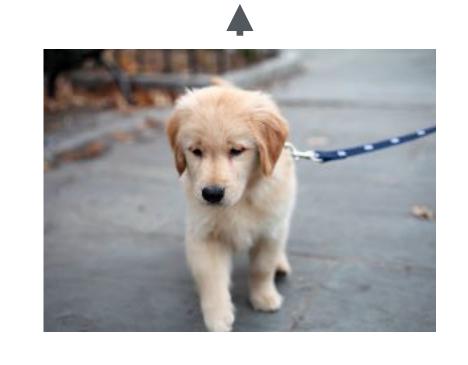
- Density estimation P(X = x)
 - . How to ensure $\sum_{x} P(x) = 1$ for all x
 - Impossible to compute (in general)
- Sampling $x \sim P(X)$
 - What is the input to the network?



Deep Network



Deep Network



Generative vs Discriminative models

Generative

- Density estimation P(X = x)
 - . How to ensure $\sum_{x} P(x) = 1$ for all x
 - Impossible to compute
- Sampling $x \sim P(X)$
 - What is the input to the network?

Discriminative

- Prediction P(Y|X)
 - Simple, explicit distribution
 - Discrete $P(y | x) = c_y^{\mathsf{T}} f(x)$
 - Continuous $P(y | x) = \mathcal{N}(y; \mu(x), \sigma(x))$
 - Well defined input y

Generative models

Two kinds of models

Sampling based $x \sim P(X)$

- Sample $z \sim P(Z)$
- Learn transformation
 - P(x|z) or $f:z \to x$

7

Deep Network



Density estimation based P(X)

- Learn special form of P(X)
- Model specific sampling / generation



Deep Network

P(X)

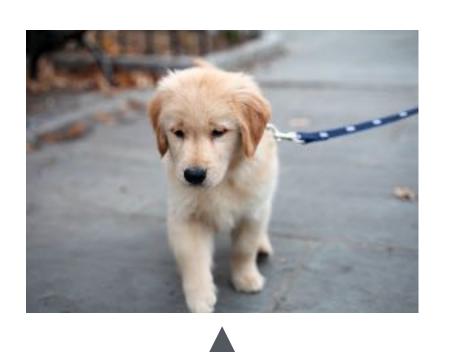
References

- [1] Dosovitskiy, Alexey, et al. "An Image is Worth 16x16 Words: Transformers for Image Recognition at Scale." International Conference on Learning Representations. 2020.
- [2] Baevski, Alexei, et al. "wav2vec 2.0: A framework for self-supervised learning of speech representations." Advances in neural information processing systems 33 (2020): 12449-12460.
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- [4] Remi Lam et al. "Learning skillful medium-range global weather forecasting." Science 382, 1416-1421(2023).
- [5] Brook, Tim, et al. "Video generation models as world simulators" OpenAI Blog (2024)
- [6] Kim, Jaehyeon, et al. "Glow-tts: A generative flow for text-to-speech via monotonic alignment search." Advances in Neural Information Processing Systems 33 (2020): 8067-8077.
- [7] Hu, Anthony, et al. "Gaia-1: A generative world model for autonomous driving." arXiv preprint arXiv:2309.17080 (2023).
- [8] J. A. Amador Herrera, et al. "Weatherscapes: Nowcasting Heat Transfer and Water Continuity." ACM Transactions on Graphics (SIGGRAPH Asia 2021), Vol. 40, No. 6, Article 204..

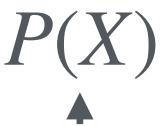
Variational Auto Encoders

Generative models

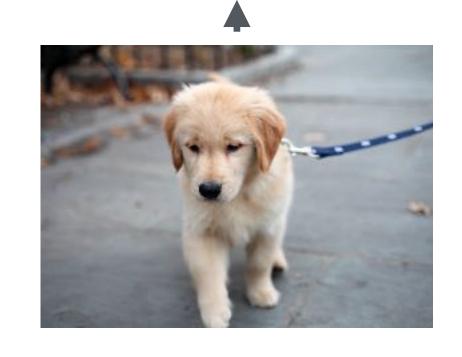
- Two tasks of a generative model P(X)
 - Sampling: $x \sim P(X)$
 - Density estimation: P(X = x)



Deep Network

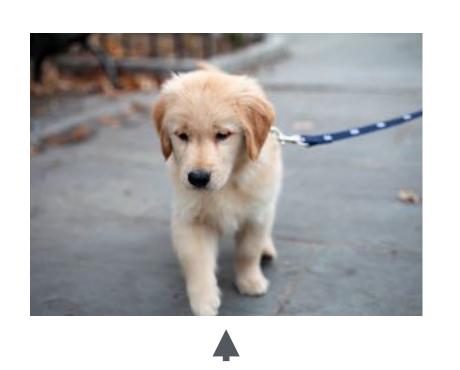


Deep Network

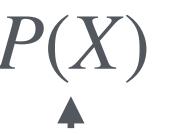


Generative modeling is hard

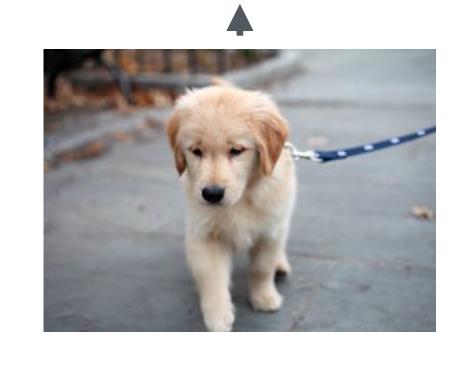
- Density estimation P(X = x)
 - . How to ensure $\sum_{x} P(x) = 1$ for all x
 - Impossible to compute (in general)
- Sampling $x \sim P(X)$
 - What is the input to the network?



Deep Network



Deep Network



Generative models

Two kinds of models

Sampling based $x \sim P(X)$

- Sample $z \sim P(Z)$
- Learn transformation
 - P(x|z) or $f:z \to x$

7

Deep Network



Density estimation based P(X)

- Learn special form of P(X)
- Model specific sampling / generation

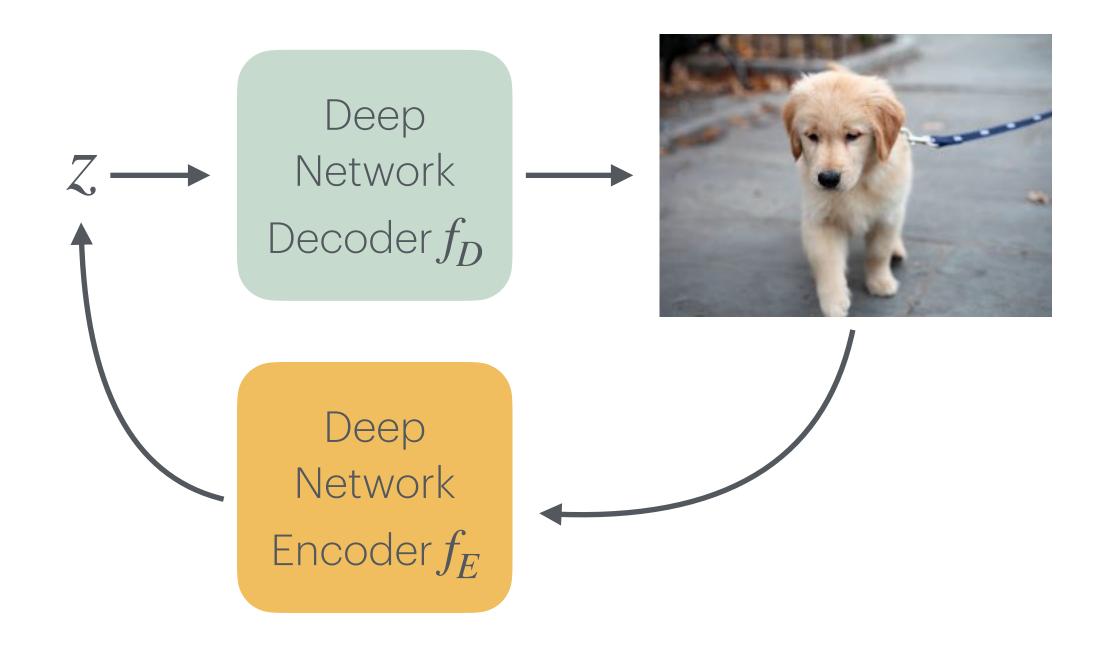


Deep Network

P(X)

Generative models

- Goal: Learn decoder $f_D: z \to x$
- What should z be?
 - Let a deep network decide
 - Encoder $f_E: x \to z$



Auto-encoder



Deep Network Encoder f_E

Deep Network Decoder f_D

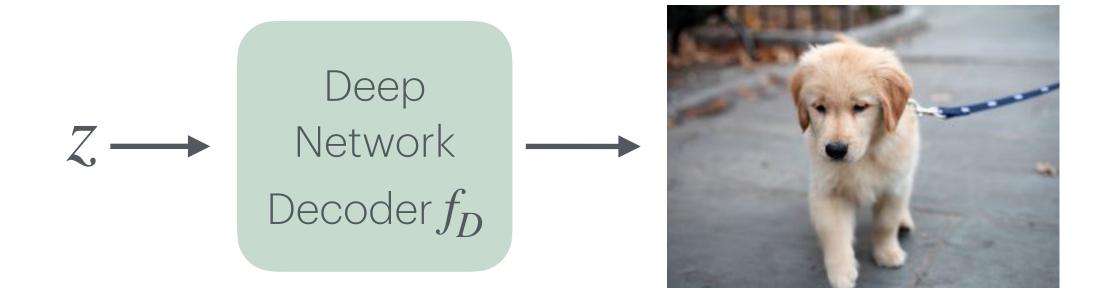
► Z ►



- encoder $z = f_E(x)$
- decoder $\hat{x} = f_D(z)$
- Training
 - Supervised learning on large dataset
 - $\mathscr{E} = E_x \left[|f_D(f_E(x)) x| \right]$

Auto-encoder as a Generative model

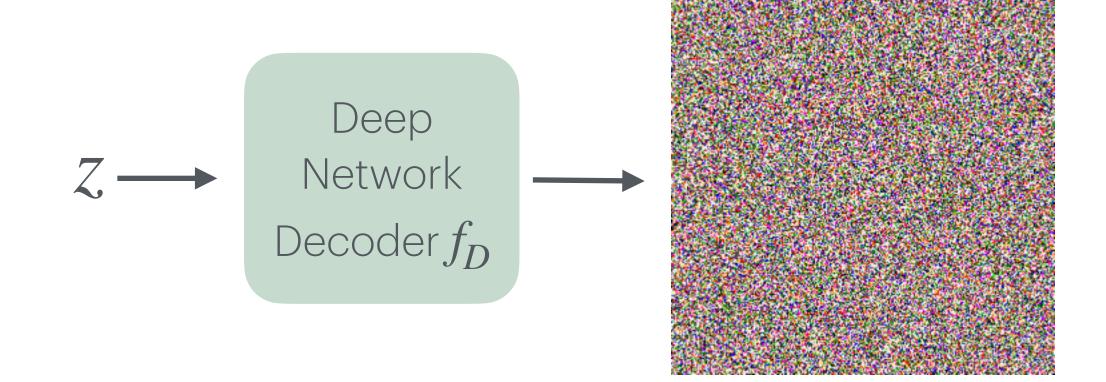
- Decoder $f_D: z \to x$
- Inference / Sampling
 - What is z at test time?



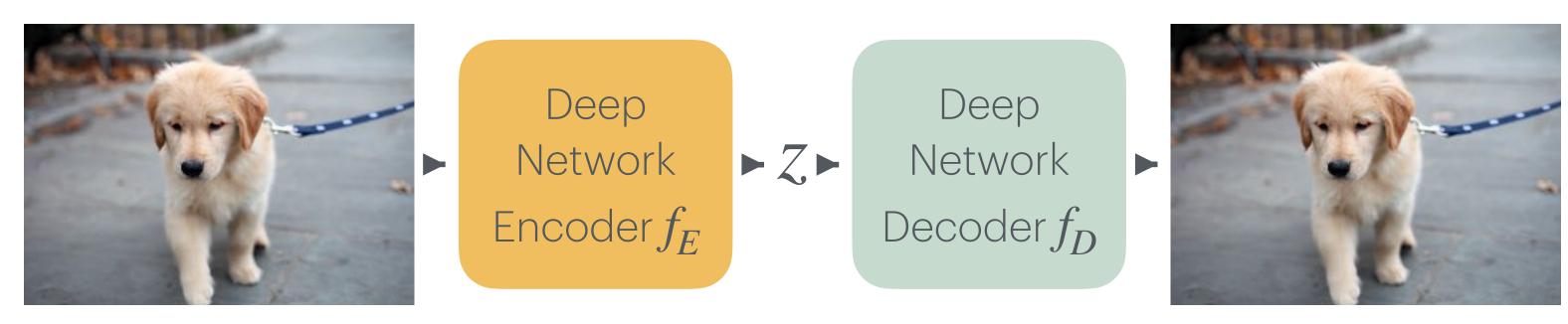
Auto-encoder

Generation

- Decoder $f_D: z \to x$
- Inference / Sampling
 - What is z at test time?
 - Network output -> no new image
 - Random input -> Garbage
 - Interpolation -> Garbage

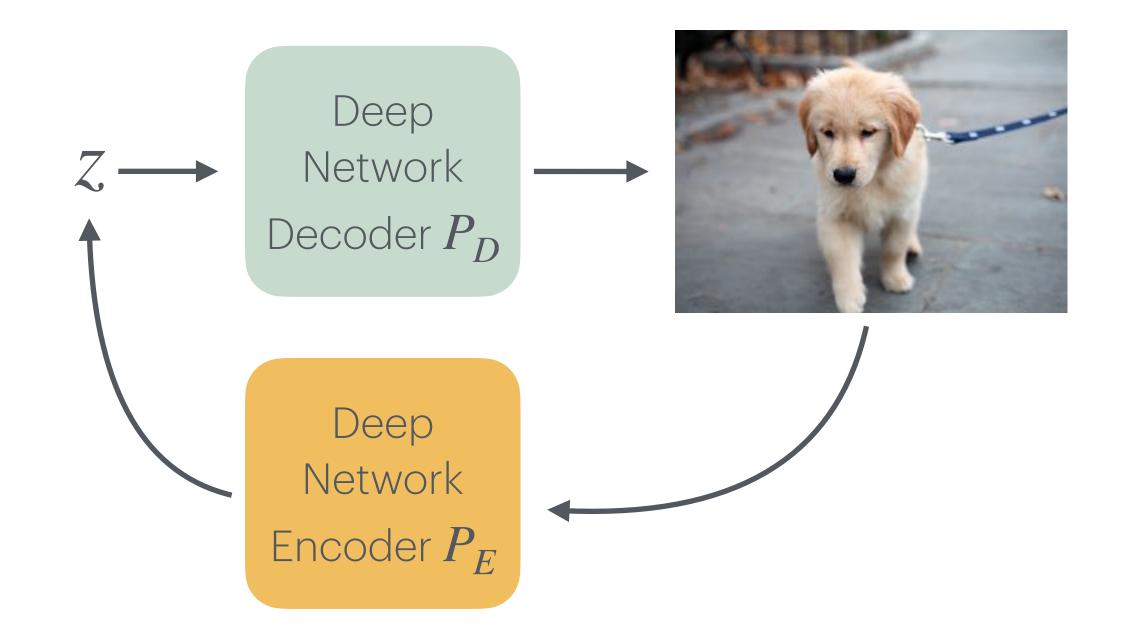


What does an auto-encoder learn?



- Compression
- "Invertible" mapping
- Does it learn the structure of images?
 - Only in the limit
 - Perfect compression = understanding
- Poor generation

- Goal: Learn decoder $P_D(x \mid z)$
- What should z be?
 - Let a deep network decide
 - Encoder $P_E(z | x)$

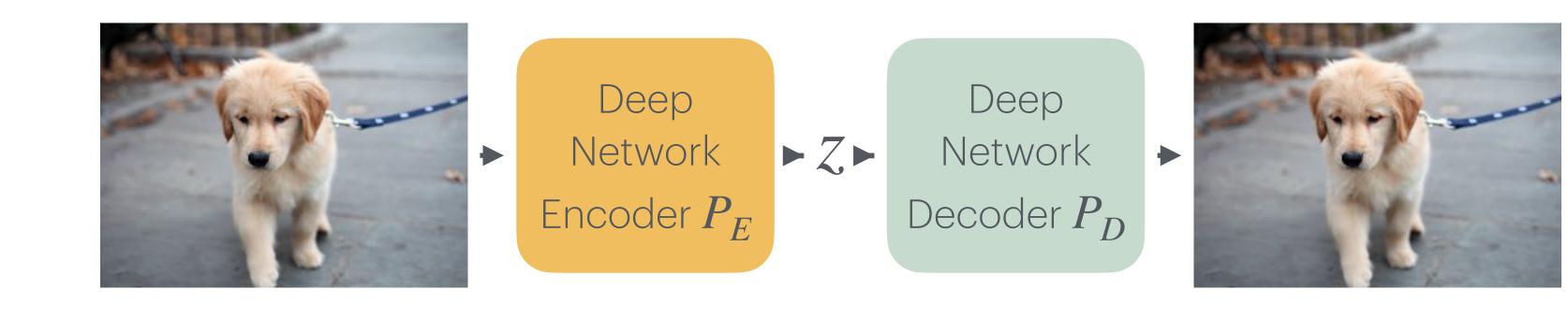


A "probabilistic" auto-encoder

- Decoder $P_D(x \mid z)$ (similar to discriminative model)
- Encoder $P_E(z \mid x)$ (similar to discriminative model)
- Assume $P(Z) = \mathcal{N}(0,1)$

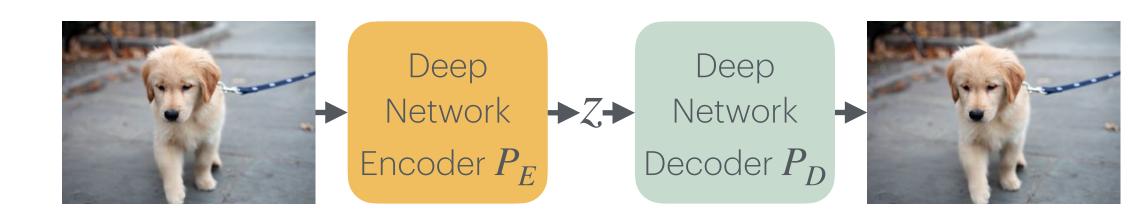
$$P(x) = \sum_{z} P_{D}(x \mid z) P(z)$$

• $z \sim P(X)$ is equivalent to $z \sim P(Z)$ and $x \sim P(x \mid z)$



- Decoder $P_D(x \mid z)$ (similar to discriminative model)
- Encoder $P_E(z \mid x)$ (similar to discriminative model)
- Assume $P(Z) = \mathcal{N}(0,1)$

. Bayes rule
$$P_E(z \mid x) = \frac{P_D(x \mid z)P(z)}{P(x)}$$
 —intractable

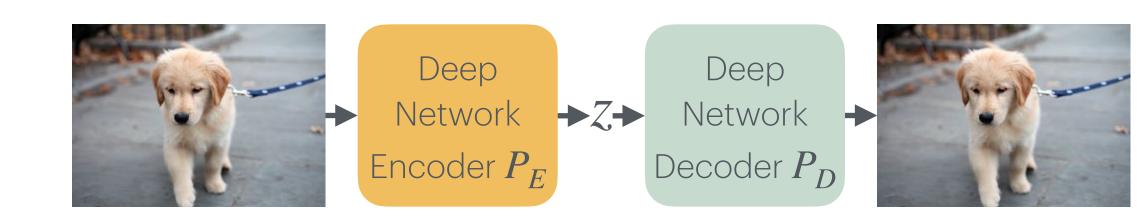


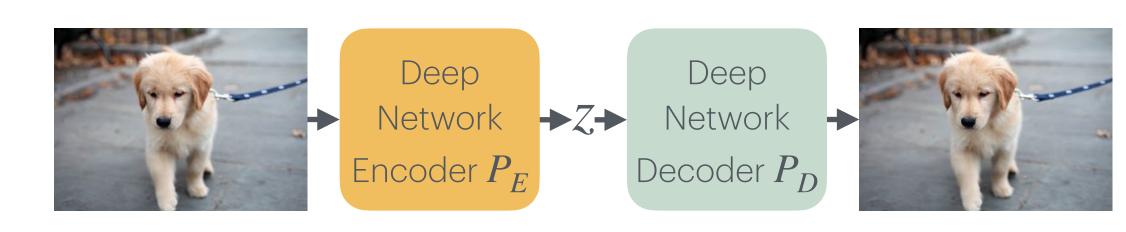
A "probabilistic" auto-encoder

- Decoder $P_D(x \mid z)$ (similar to discriminative model)
- Encoder Q(z | x) (similar to discriminative model)
- Assume $P(Z) = \mathcal{N}(0,1)$

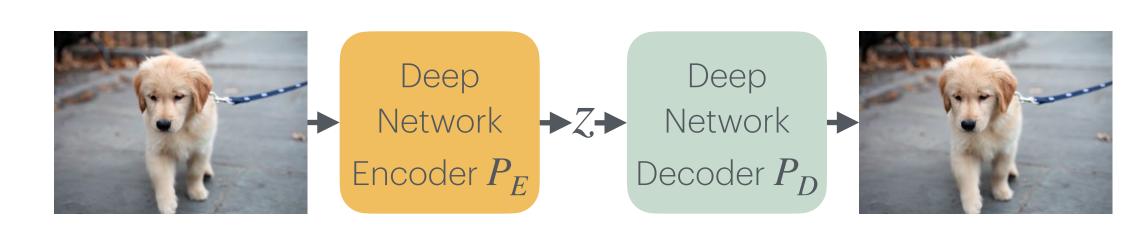
Bayes rule
$$P_E(z \mid x) = \frac{P_D(x \mid z)P(z)}{P(x)}$$
 —intractable

• Learn $Q pprox P_E$ that minimizes $D_{\mathit{KL}}(Q \,|\, P_E)$





- Learn $Q \approx P_E$ that minimizes $D_{\mathit{KL}}(Q(z\,|\,x)||P_E(z\,|\,x)) = \log P(x) + E_{z\sim Q} \left[\log \frac{P(z)P_D(x\,|\,z)}{Q(z\,|\,x)}\right]$
- Maximize $\log P(x)$ of real data, minimize D_{KL} $\log P(x) D_{KL}(Q(z \mid x) || P_E(z \mid x)) = E_{z \sim Q} \left[\log \frac{Q(z \mid x)}{P(z) P_D(x \mid z)} \right]$
 - Known as ELBO (Evidence Lower BOund)

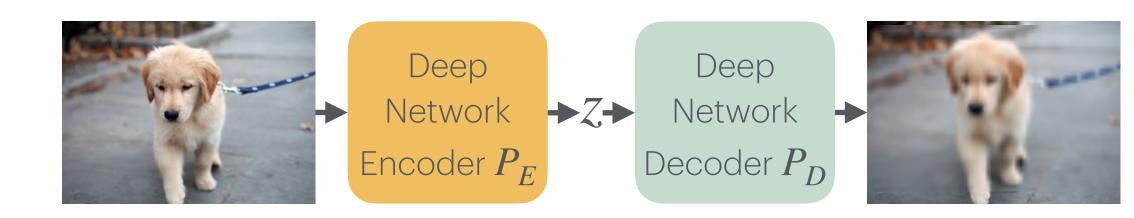


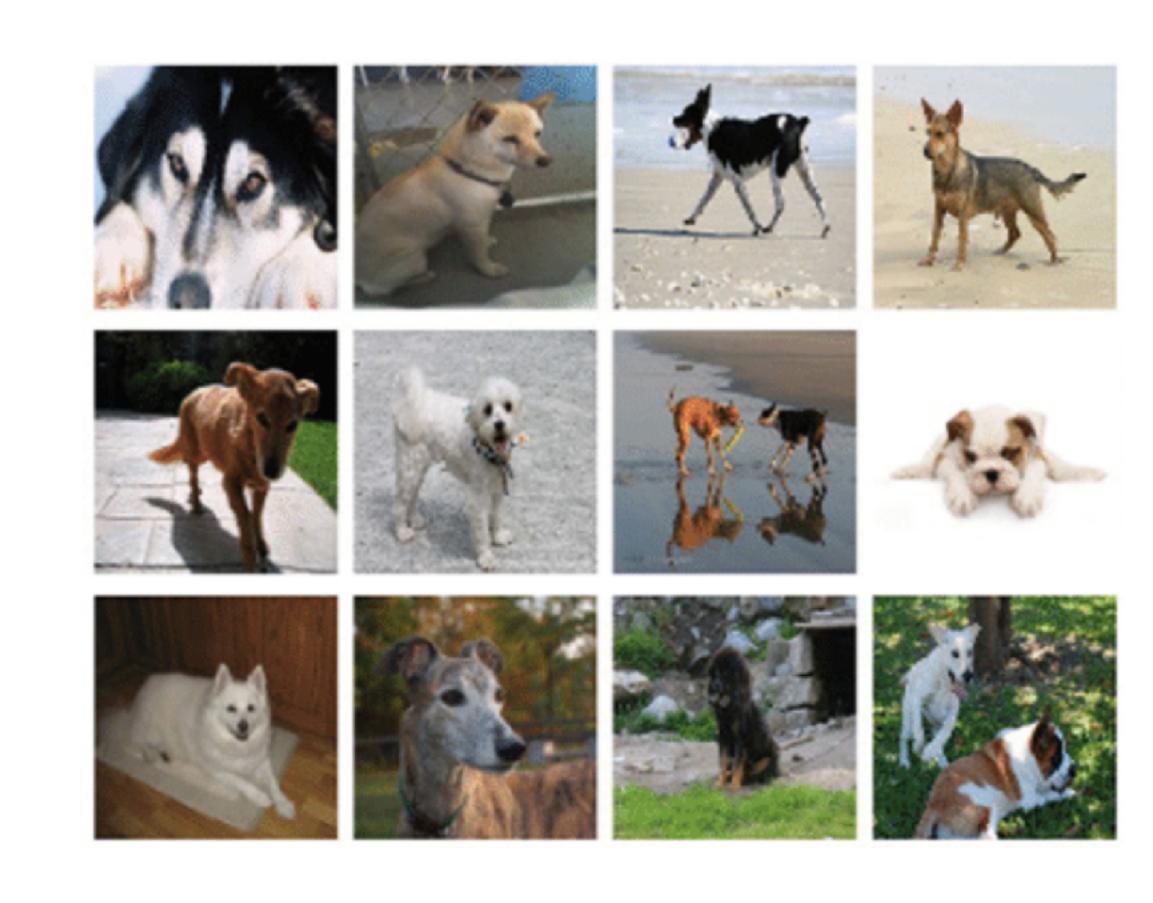
. ELBO
$$E_{z\sim Q}\left[\log \frac{Q(z\,|\,x)}{P(z)P_D(x\,|\,z)}\right]$$
 for Gaussians

•
$$-\frac{1}{2}\mathbb{E}_{z\sim Q}\left[\|x-\mu_D(z)\|_2^2\right] - \frac{1}{2}\left(N\sigma_Q(x)^2 + \|\mu_Q(x)\|_2^2 - 2N\log\sigma_Q(x)\right) + Const$$

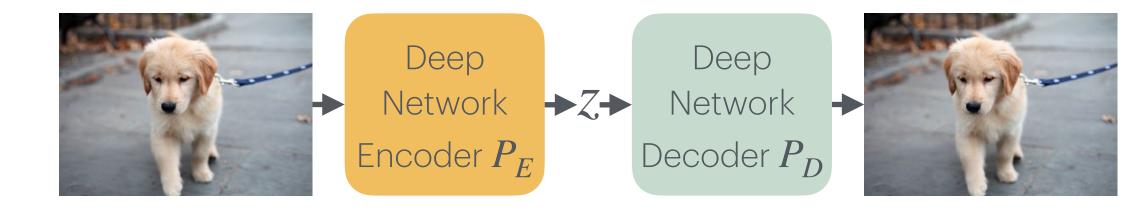
- Reparametrization trick
 - . For $Q(z \mid x) = \mathcal{N}(z; \mu_Q(x), \sigma_Q^2(x))$
 - $\mathbb{E}_{z \sim Q} \left[\|x \mu_D(z)\|_2^2 \right] = \mathbb{E}_{\varepsilon \sim \mathcal{N}(0,1)} \left[\|x \mu_D(\mu_Q(x) + \varepsilon \sigma_Q(x))\|_2^2 \right]$

- Can learn P(X) and sampling function $x \sim P$
- Issues
 - Reconstruction loss: Pixel-level | 2 loss
 - Blurry outputs
 - Approximation Q: Gaussian assumption
 - Sphere packing in higher dimensions
 - Lots of empty space





- Learn a model of $P(x) = P_D(x \mid z)P(z)$ with $P(z) = \mathcal{N}(z; 0, 1)$
 - Training: Maximize P(x) of data
 - Approximate $Q pprox P_E$





References

• [1] Auto-Encoding Variational Bayes. Kingma et al. 2014.

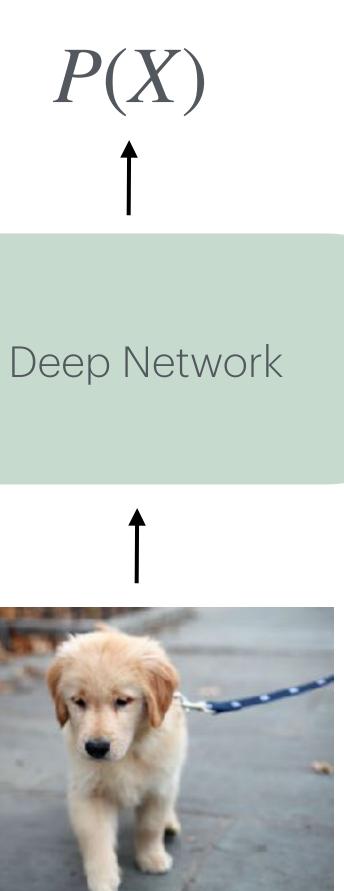
Auto-regressive generation

Generative models

- Two tasks of a generative model P(X)
 - Sampling: $x \sim P(X)$
 - Density estimation: P(X = x)



Deep Network

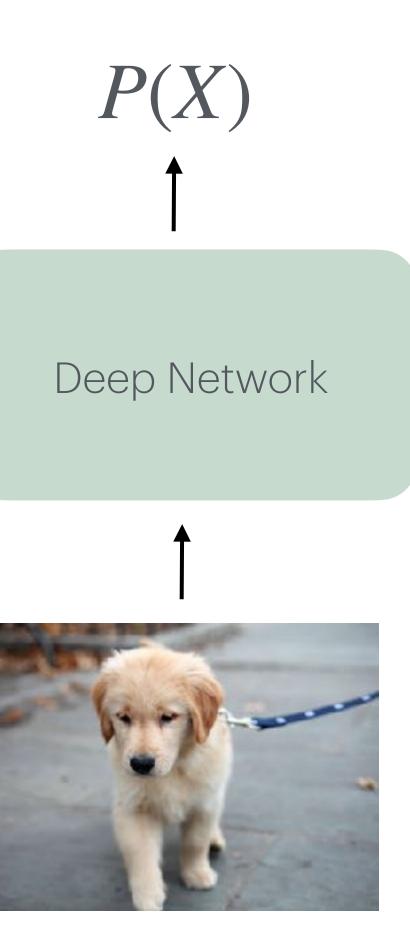


Generative modeling is hard

- Density estimation P(X = x)
 - . How to ensure $\sum_{x} P(x) = 1$ for all x
 - Impossible to compute (in general)
- Sampling $x \sim P(X)$
 - What is the input to the network?



Deep Network



Generative models

Two kinds of models

Sampling based $x \sim P(X)$

- Sample $z \sim P(Z)$
- Learn transformation
 - P(x|z) or $f:z \to x$

7

Deep Network



Density estimation based P(X)

- Learn special form of P(X)
- Model specific sampling / generation



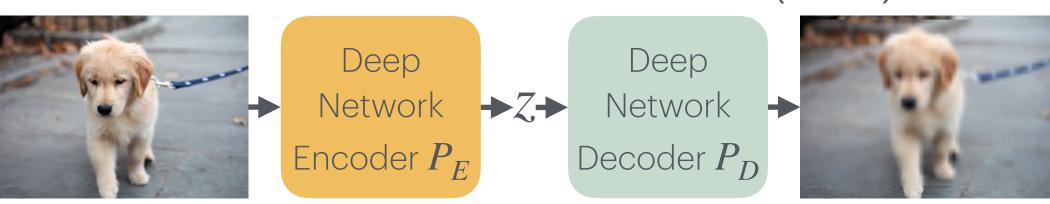
Deep Network

P(X)

Recap

- VAE
 - Image -> latent space -> Image
 - Loss encourages Gaussian latent

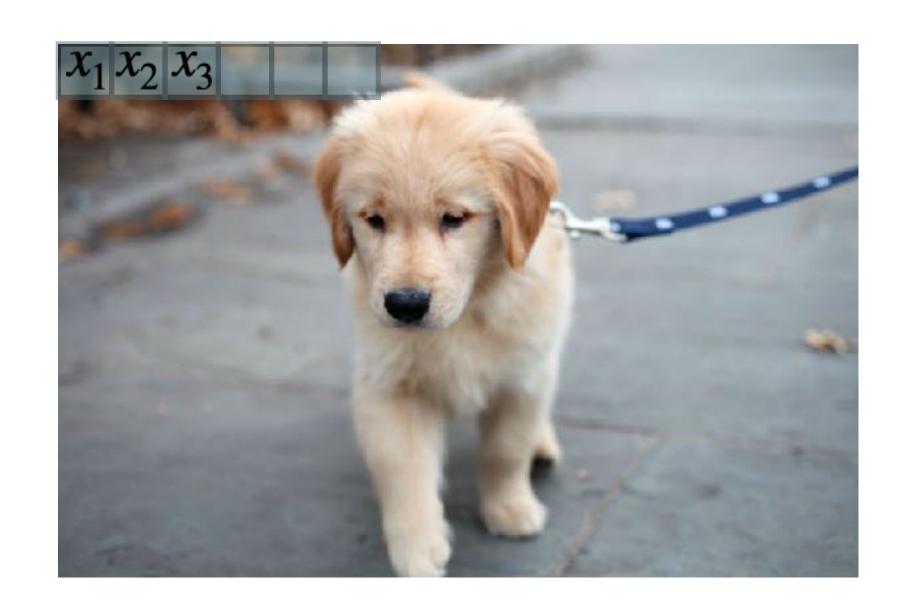
Variational Auto Encoder (VAE)



Auto-regressive models

$$P(x) = P(x_1)P(x_2 | x_1)P(x_3 | x_1, x_2)P(x_4 | x_1...x_3)...$$

- $P(x_i | x_1...x_{i-1}) = \text{softmax}(f(x_1...x_{i-1}))$
- Basis of most LLM models
- Easy estimation of P(x)
- Easy sampling $x_1 \sim P(X_1); x_2 \sim P(X_2 \mid x_1)$
 - Slow sampling



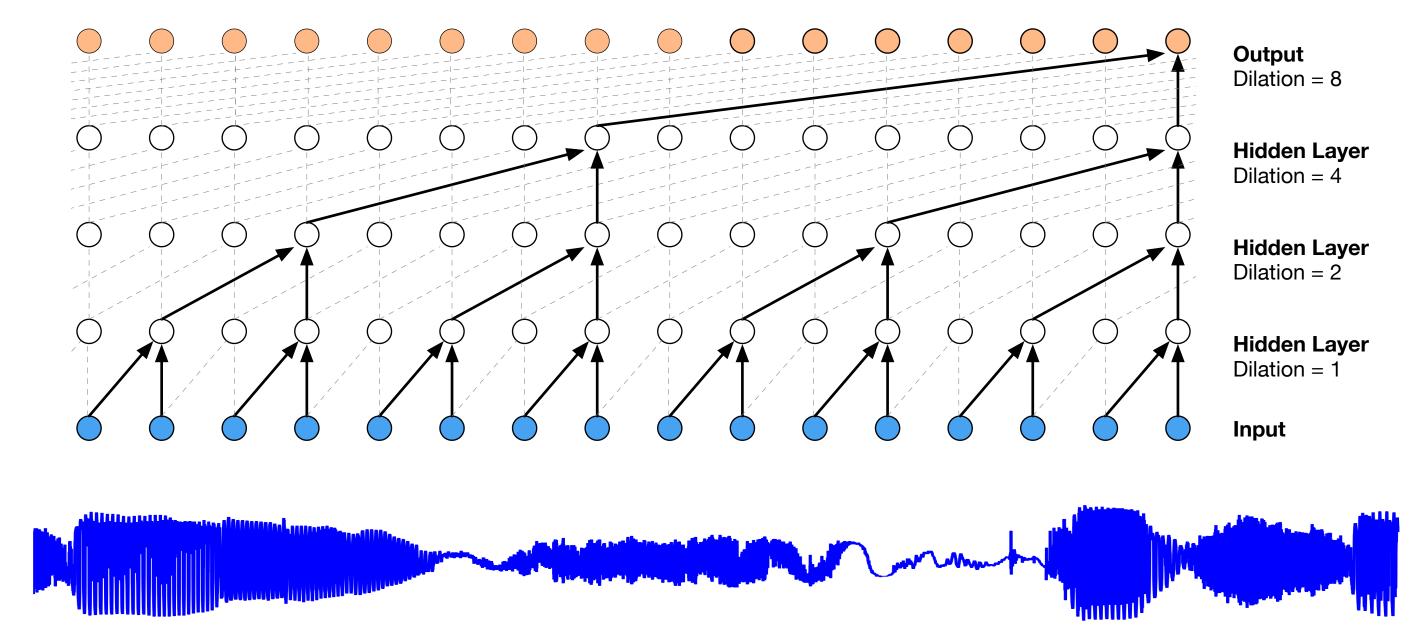
Example: WaveNet

- Input: Raw waveform $\mathbf{x}_{1...t-1}$
- Output: Quantized next value $\mathbf{x}_t \in \{1...256\}$

Model:
$$P(\mathbf{x}) = \prod_{t=1}^{T} P(x_t | \mathbf{x}_{1...t-1})$$

Conditioned model:

$$P(\mathbf{x} \mid \mathbf{h}) = \prod_{t=1}^{I} P(x_t \mid \mathbf{x}_{1...t-1} \mid \mathbf{h})$$



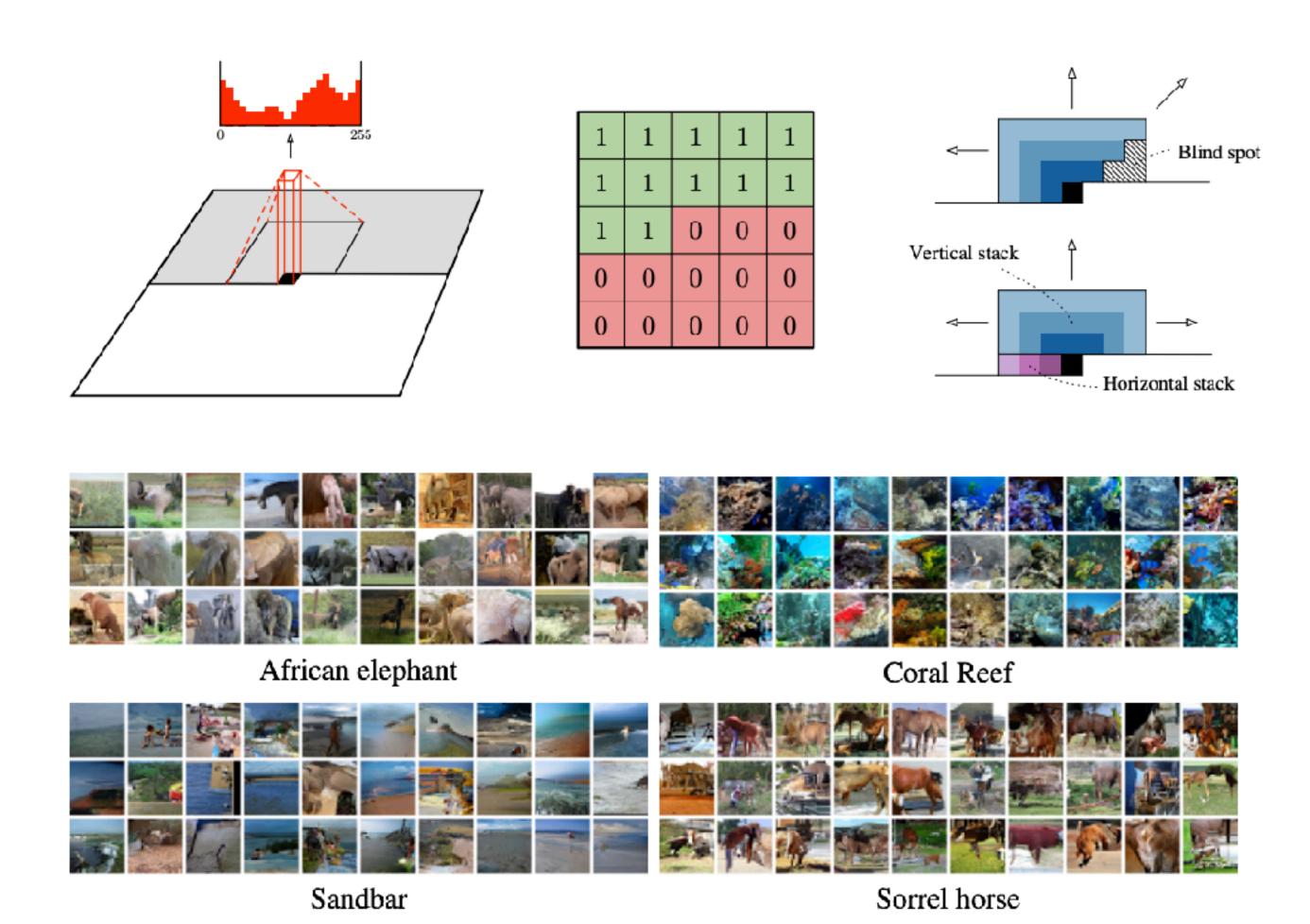
Example: PixelCNN

- Input: Raw pixels $\mathbf{x}_{1...t-1}$
- Output: Quantized next color value $\mathbf{x}_t \in \{1...256\}$

Model:
$$P(\mathbf{x}) = \prod_{t=1}^{T} P(x_t | \mathbf{x}_{1...t-1})$$

Conditioned model:

$$P(\mathbf{x} \mid \mathbf{h}) = \prod_{t=1}^{I} P(x_t \mid \mathbf{x}_{1...t-1} \mid \mathbf{h})$$

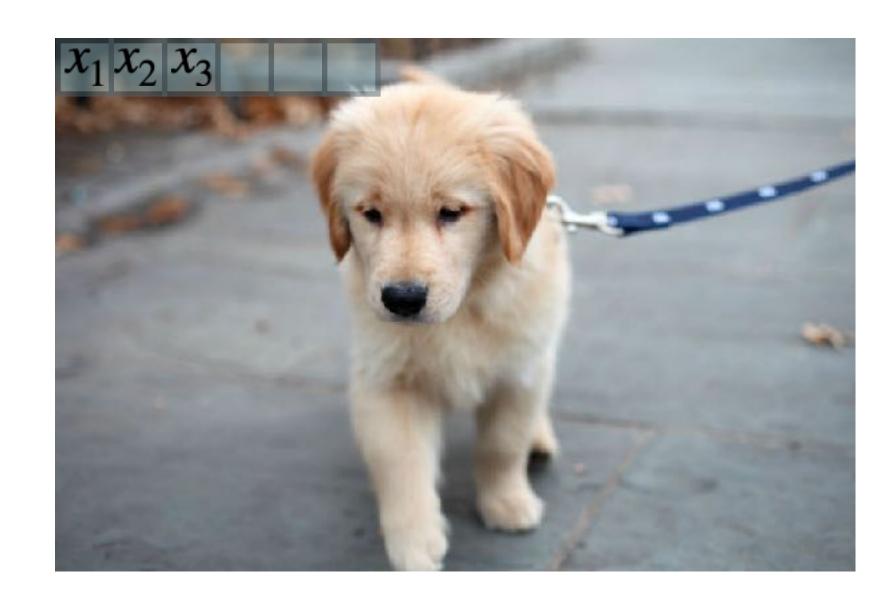


Auto-regressive models

Issues

$$P(x) = P(x_1)P(x_2 | x_1)P(x_3 | x_1, x_2)P(x_4 | x_1...x_3)...$$

- Difficult learning problem for long sequences (requires good model)
- Solution: Tokenization/Vector-Quantization (next class)
 - More complex x_i
 - Shorter sequence



Generation vs Compression

• Knowing $P(\mathbf{x})$ leads to best lossless compression within one bit

• #bits =
$$\left[-\log_2 P(\mathbf{x})\right] + 1$$

Why?

Arithmetic coding

 $[-\log_2 P(\mathbf{x})] + 1$ bit lossless compression

- Sort **x** lexicographically
 - Compute CDF P(X < x)
 - Split interval between 0...1 into $2^{\lfloor -\log_2 P(\mathbf{x}) \rfloor + 1}$ numbers
 - . Since $2^{\lfloor -\log_2 P(\mathbf{x}) \rfloor + 1} > \frac{1}{P(\mathbf{x})}$, at least one number n will end in range $P(\mathbf{X} < \mathbf{x}) \dots P(\mathbf{X} \le \mathbf{x})$
 - n is our $\left[-\log_2 P(\mathbf{x})\right] + 1$ code

P(X < x) $P(\mathbf{x})$

[1] Lossless Image Compression through Super-Resolution. Sheng Cao, et al. 2020

[2] Practical Full Resolution Learned Lossless Image Compression. Fabian Mentzer, et al. 2019

Arithmetic coding in practice

• CDF $P(\mathbf{X} < \mathbf{x})$ generally hard to compute

Easy for
$$P(\mathbf{x}) = \prod_{t=1}^{T} P(x_t | \mathbf{x}_{1...t-1})$$

$$P(\mathbf{X} \le \mathbf{x}) = \prod_{t=1}^{T} P(X_t \le x_t | \mathbf{x}_{1...t-1})$$

Leads to adaptive arithmetic coding

Generative models

Two kinds of models

Sampling based $x \sim P(X)$

- Sample $z \sim P(Z)$
- Learn transformation
 - P(x|z) or $f:z \to x$

7

Deep Network



Density estimation based P(X)

- Learn special form of P(X)
- Model specific sampling / generation



Deep Network

P(X)

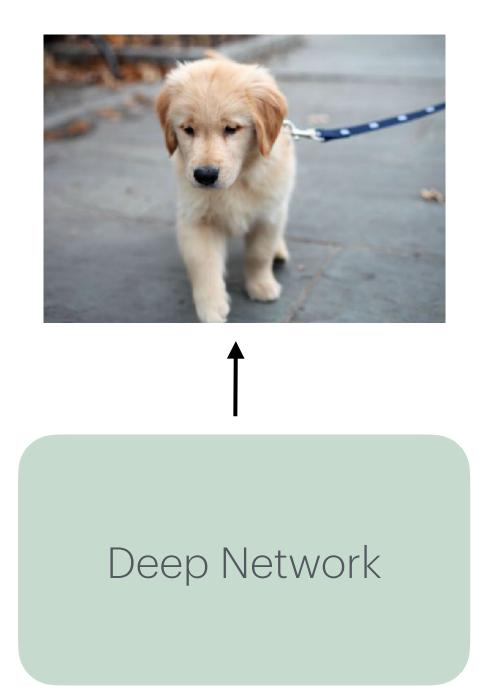
References

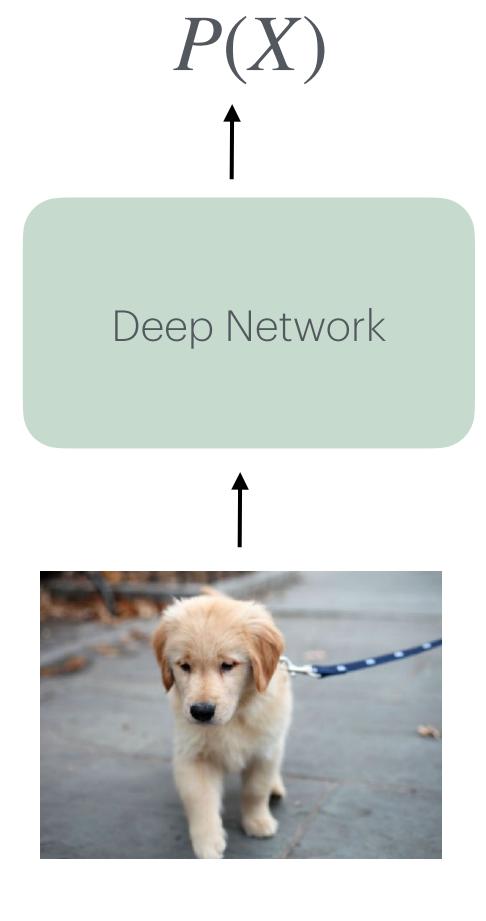
- [1] WaveNet: A Generative Model for Raw Audio. Aaron van den Oord, et al. 2016
- [2] Long Video Generation with Time-Agnostic VQGAN and Time-Sensitive Transformer. Songwei Ge, et al. 2022
- [3] Lossless Image Compression through Super-Resolution. Sheng Cao, et al. 2020
- [4] Practical Full Resolution Learned Lossless Image Compression. Fabian Mentzer, et al. 2019

Vector Quantization

Generative models

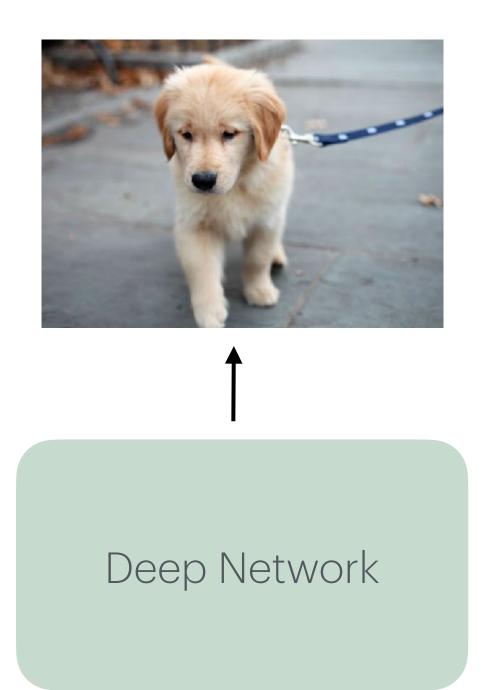
- Two tasks of a generative model P(X)
 - Sampling: $x \sim P(X)$
 - Density estimation: P(X = x)

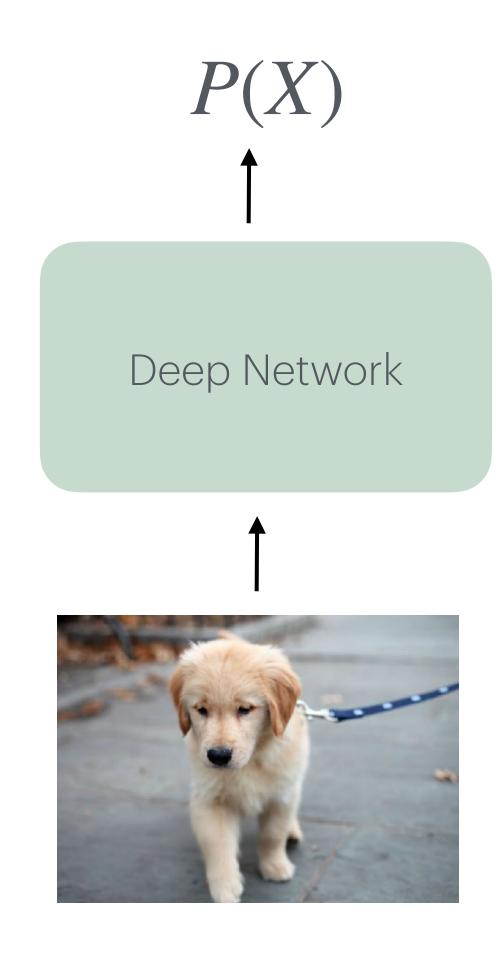




Generative modeling is hard

- Density estimation P(X = x)
 - . How to ensure $\sum_{x} P(x) = 1$ for all x
 - Impossible to compute (in general)
- Sampling $x \sim P(X)$
 - What is the input to the network?





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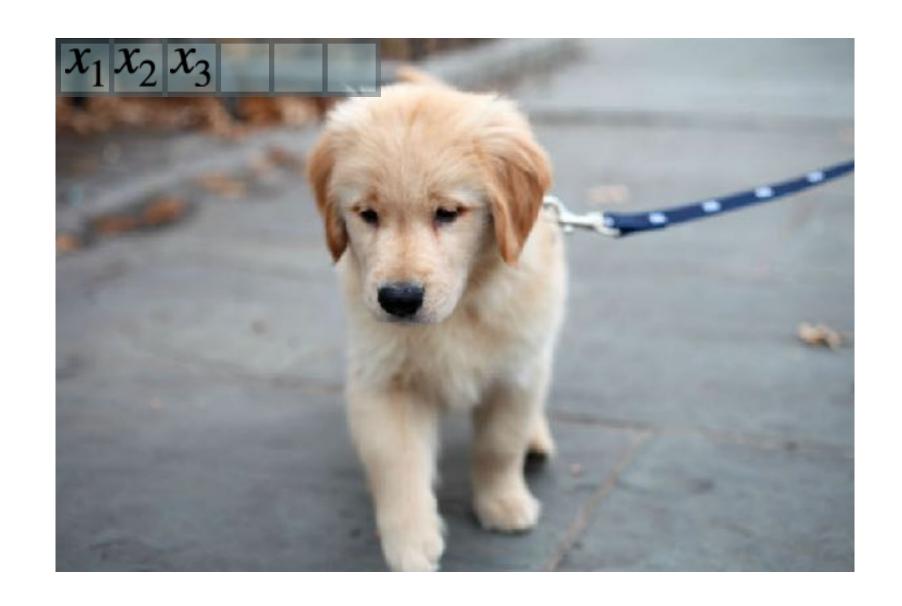
P(X)

Auto-regressive models

Issues

$$P(x) = P(x_1)P(x_2 | x_1)P(x_3 | x_1, x_2)P(x_4 | x_1...x_3)...$$

• Difficult learning problem for long sequences (requires good model)

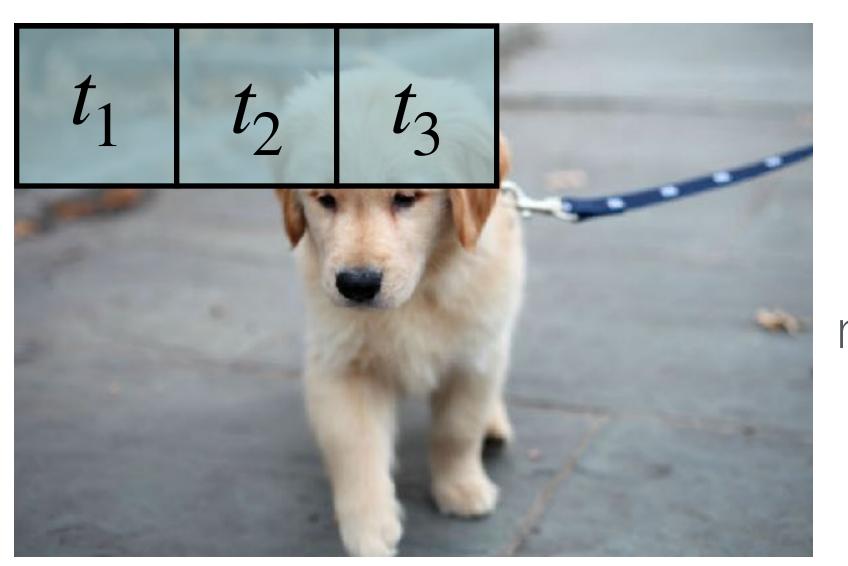


Tokenization

- Image [1]
 - Convert patch p_i of pixels into token $t_i \in \{1, \ldots, K\}$
- Text [2]
 - Convert set of characters into token
- Protein-sequence [3]
 - Convert local protein structure to token



Vanilla autoregressive model



Tokenized autoregressive model

^[1] Neural Discrete Representation Learning. Aaron van den Oord, et al. 2017

^[2] Language models are unsupervised multitask learners. Alec Radford, et al. 2019

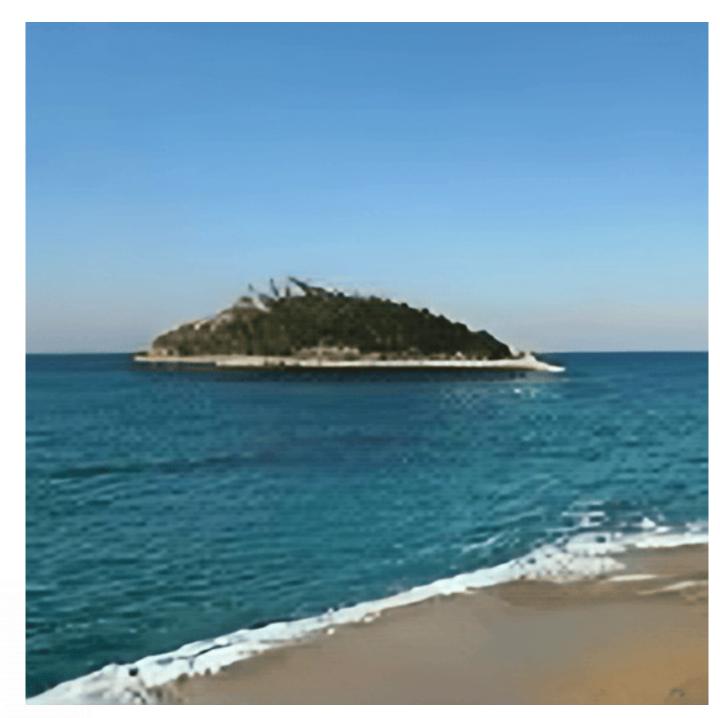
^[3] Simulating 500 million years of evolution with a language model. Thomas Hayes, et al. 2024

Auto-regressive models on tokens

$$P(\mathbf{t}) = P(t_1)P(t_2 | t_1)P(t_3 | t_1, t_2)P(t_4 | t_1...t_3)...$$

• Shorter sequence = easier to learn structure

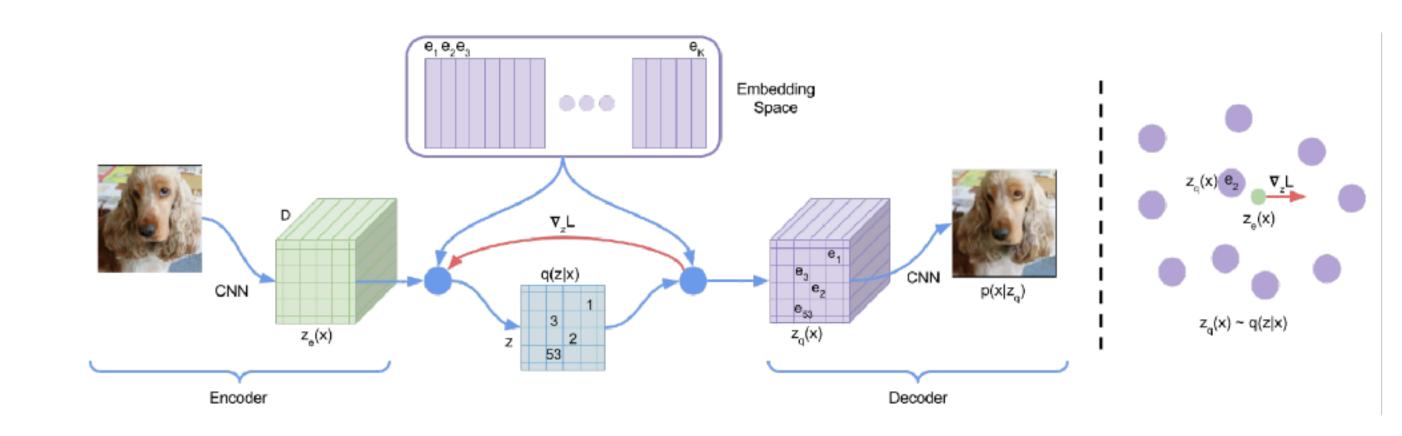




Learning Tokenization

Vector Quantization

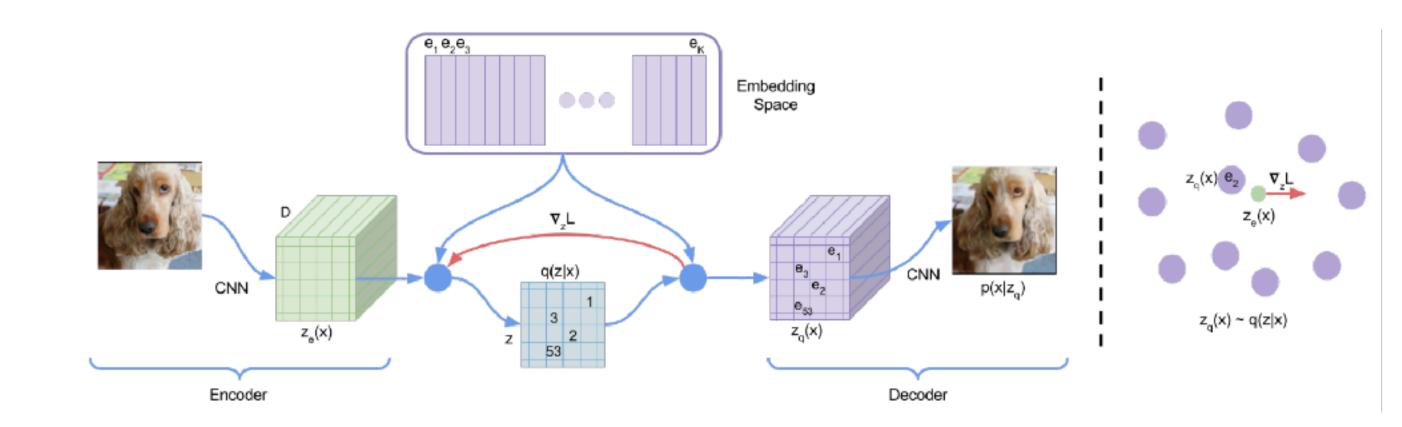
- Input: Image (or patch) $x \in \mathbb{R}^{H \times W \times 3}$
- Output: "Image" of tokens $z \in \{1...K\}^{h \times w}$
- Why is this hard to learn?
 - $z \rightarrow x$ (easy, reconstruction)
 - $x \rightarrow z \rightarrow x$ (hard, z is discrete and non-differentiable)



- Variational Auto-Encoder
 - Decoder $P_D(x \mid z)$ Encoder $Q(z \mid x)$
- Vector Quantizer

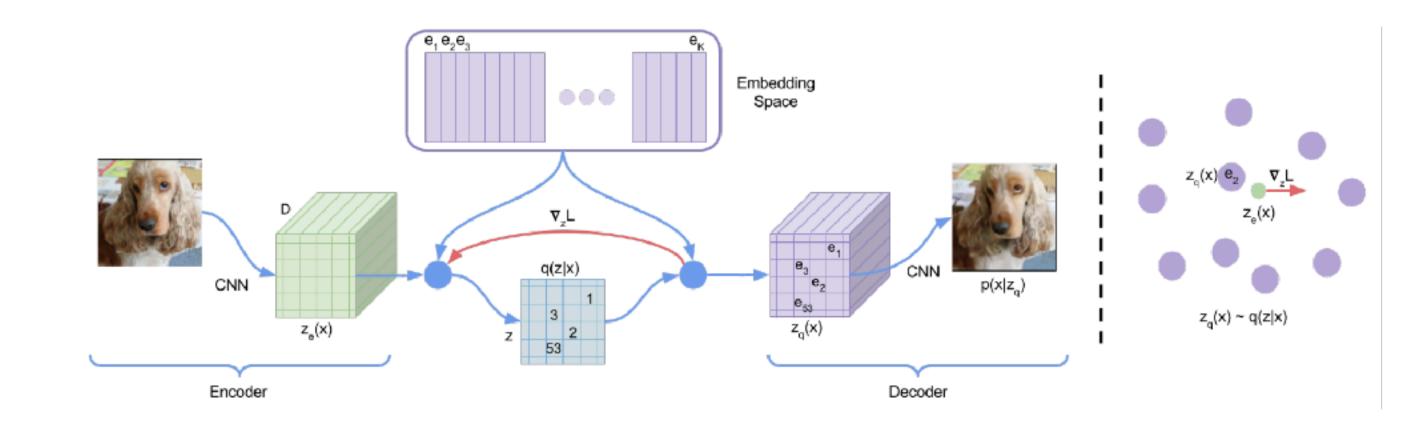
$$q(z) = \arg\min_{e_k} ||z - e_k||$$

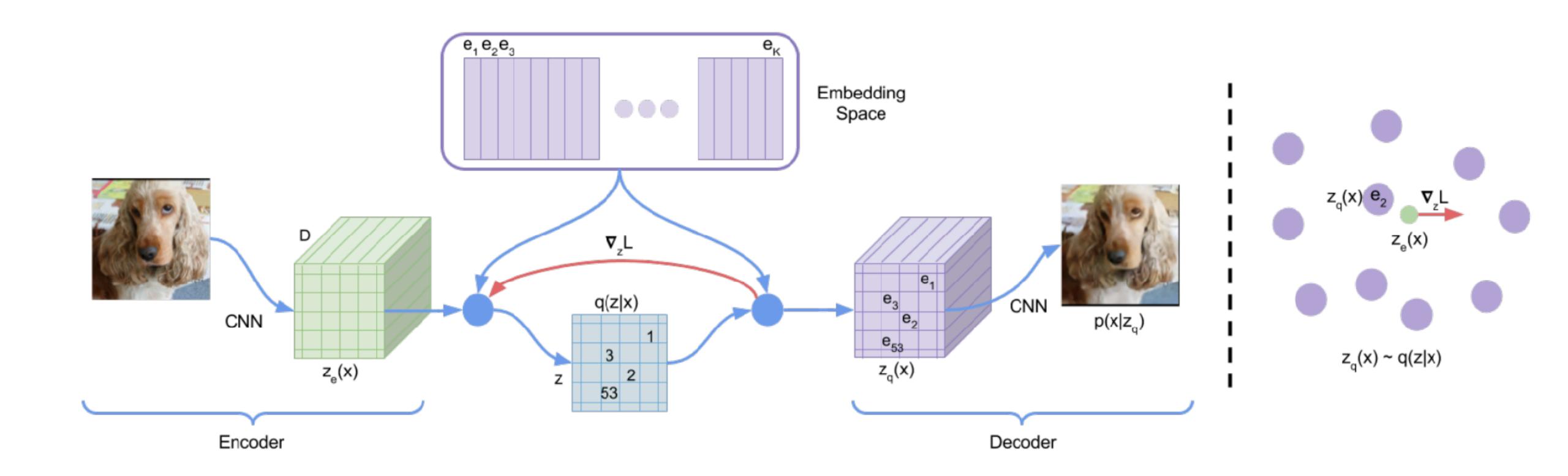
- Learn codebook $\{e_1...e_K\}$
- What is $\nabla q(z)$?



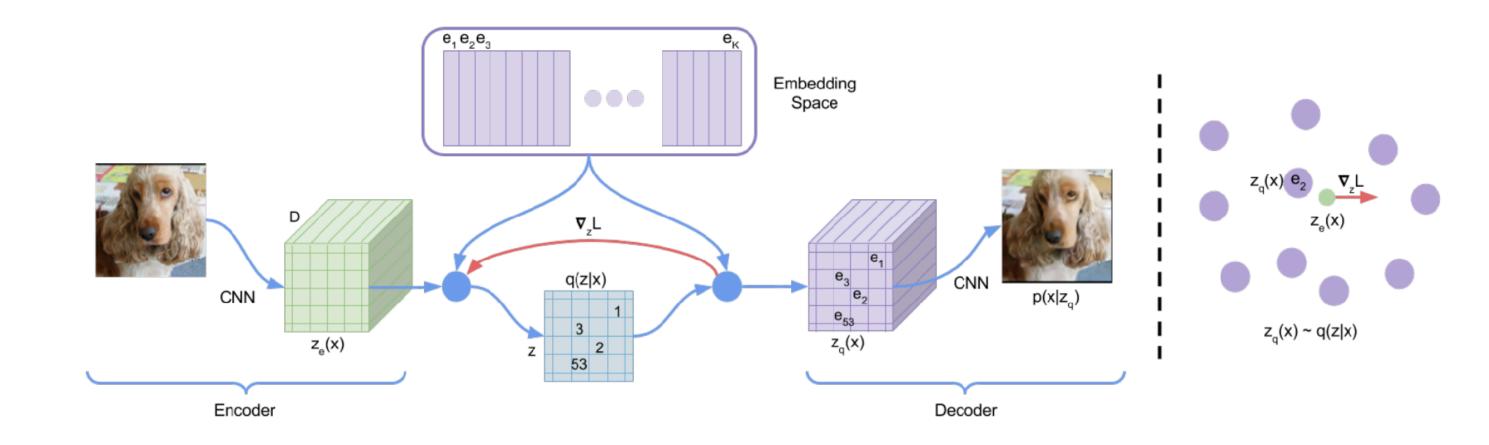
Gradient

- What is $\nabla q(z)$?
 - Let's assume $\nabla q(z) = I$ (identity)
 - Straight-Through Estimator
 - Works in practice because errors average out over large enough batches
 - No reason it should work





- Only as good as VAE
- Does not scale well with codebook size
 - Codebook grows exponentially in #bits
 - Many entries → sparse gradients
 - Slow

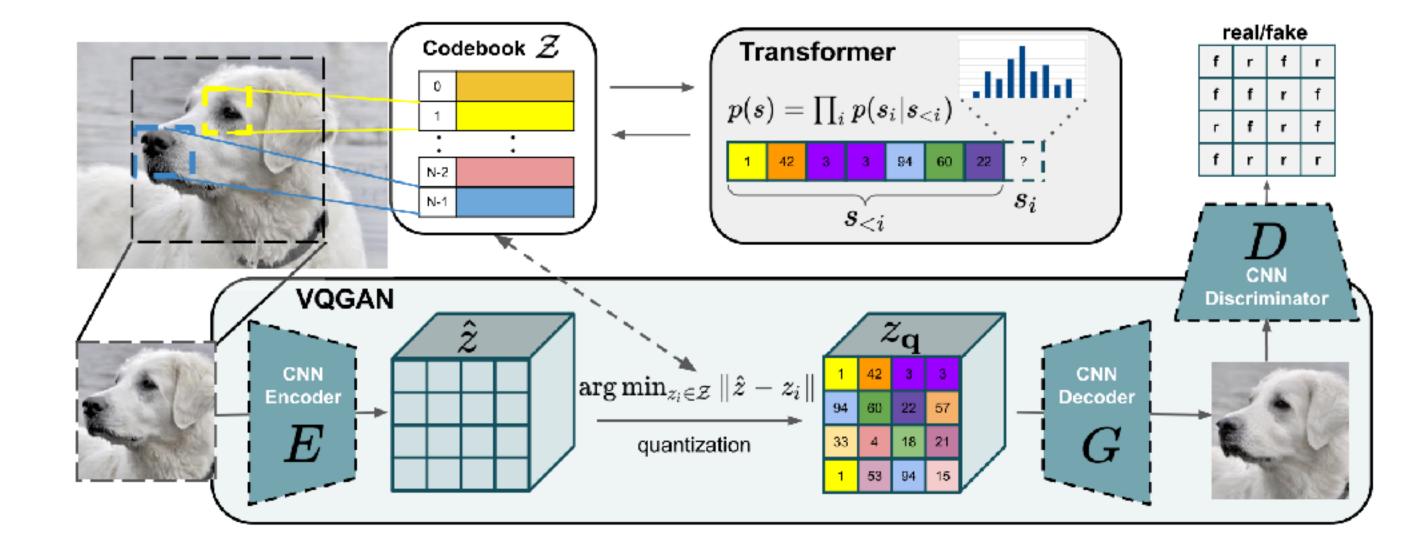


VQ-GAN

- Replace VAE with GAN
 - Auto-encoder with vector quantization

$$q(z) = \underset{e_k}{\arg\min} \|z - e_k\|$$

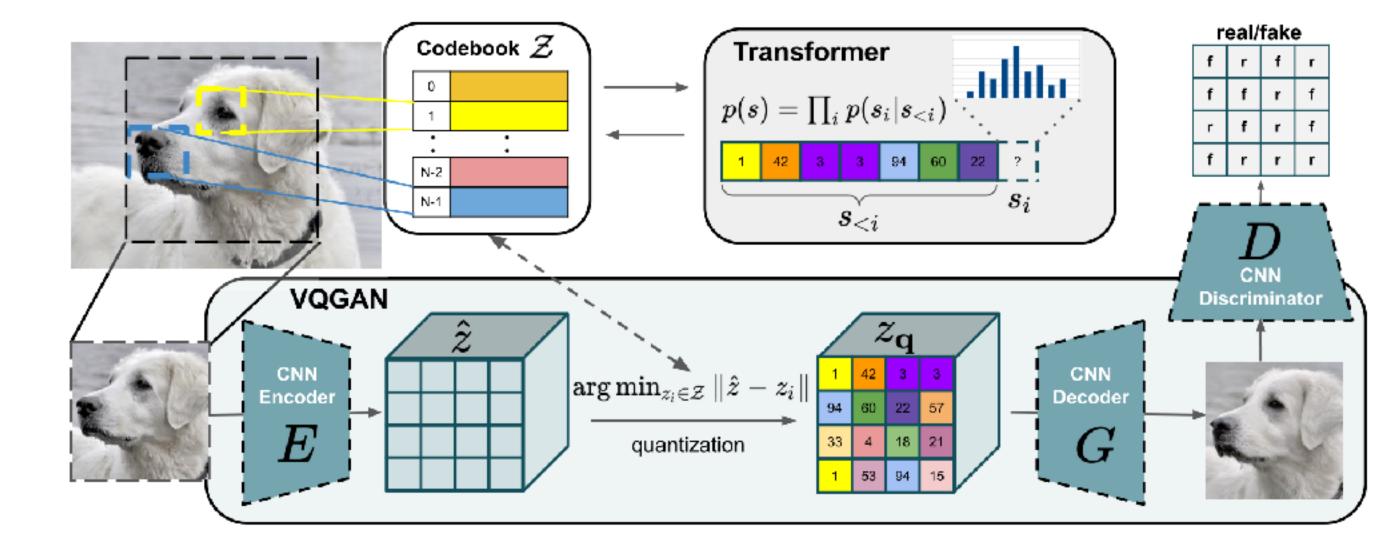
- GAN + Reconstruction loss
- Learn a sequence model on top
- Default image tokenizer nowadays





VQ-GAN

- Great tokenizer, ok sequence model
- Does not scale well with codebook size
 - Codebook grows exponentially in #bits
 - Many entries → sparse gradients
 - Slow

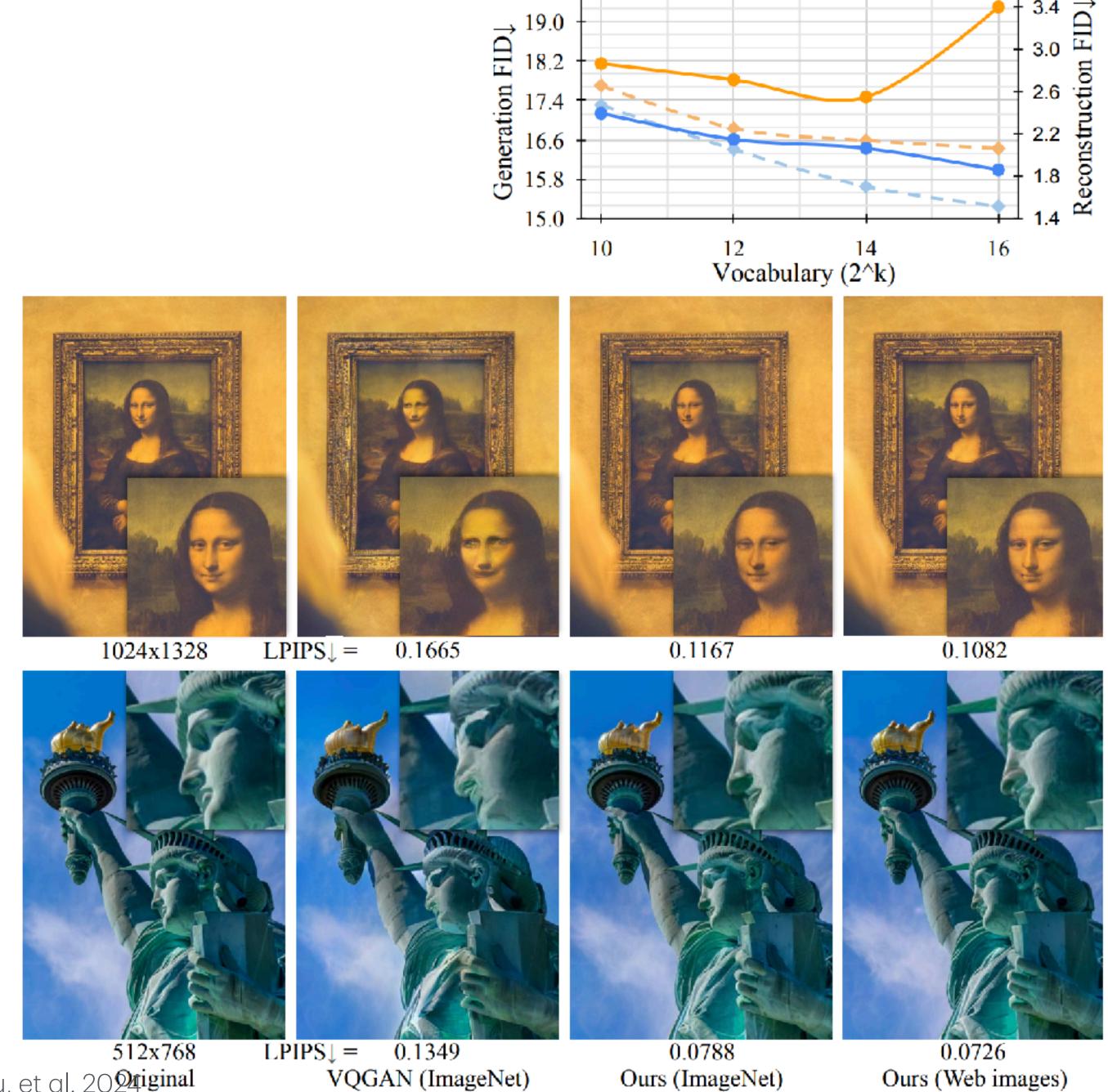




LFQ

Lookup-Free Quantization

- Different quantizer
 - q(z) = sign(z) where $sign(z_i) = 1_{[z_i \le 0]} - 1_{[z_i > 0]}$
- Scales linearly with #bits in bottleneck
- No learned parameters

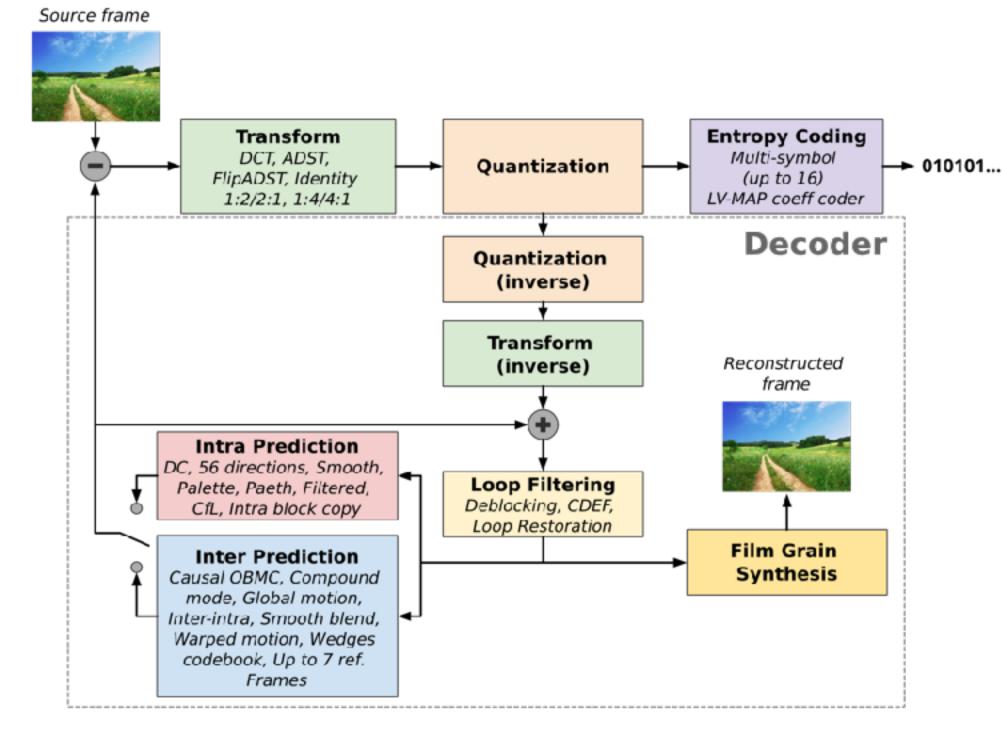


VQ Reconstruction
VQ Generation

LFQ Reconstruction
LFQ Generation

Generation vs Compression

- Auto-regressive model
 - Lossless compression (fancy gzip)
- Tokenization (VQ)
 - Lossy compression
- Similar to how JPEG most video codecs work



Source: https://commons.wikimedia.org/wiki/File:The_Technology_Inside_Av1.svg

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P(X)

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- [4] Language models are unsupervised multitask learners. Alec Radford, et al. 2019
- [5] Simulating 500 million years of evolution with a language model. Thomas Hayes, et al. 2024
- [6] MAGVIT: Masked Generative Video Transformer. Lijun Yu, et al. 2023
- [7] Estimating or Propagating Gradients Through Stochastic Neurons for Conditional Computation. Yoshua Bengio, et al. 2013
- [8] Taming transformers for high-resolution image synthesis. Patrick Esser et al. 2021
- [9] Language Model Beats Diffusion -- Tokenizer is Key to Visual Generation. Lijun Yu, et al. 2024