Deep Networks

Recap: Linear Binary Classification

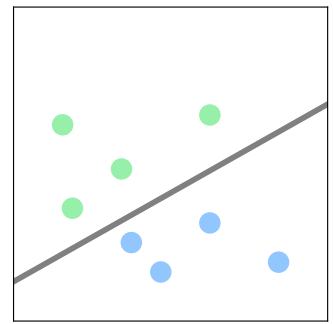
Binary classification model:

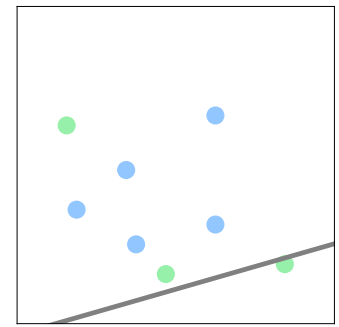
$$f_\theta : \mathbb{R}^n \to [0, 1]$$

Linear binary classification:

$$f_\theta(\mathbf{x}) = \sigma(\mathbf{W}\mathbf{x} + \mathbf{b})$$

$$\sigma(x) = \frac{1}{1 + e^{-x}}$$





Limitations of Linear Models

Binary paw classification

- Dog paw or not

Linear models

A linear model cannot distinguish paws from background

Why Does the Linear Model Break?

By linearity,

$$\mathbf{W}^T\mathbf{x}_1 + \mathbf{b} > 0 \qquad \mathbf{x}_1 = (\text{white paw})$$

$$\mathbf{W}^T\mathbf{x}_2 + \mathbf{b} > 0 \qquad \mathbf{x}_2 = (\text{black paw})$$

Then, $\mathbf{W}^T\mathbf{x} + \mathbf{b} > 0$

- for any $\mathbf{x} = \alpha\mathbf{x}_1 + (1-\alpha)\mathbf{x}_2$
- gray background $= \frac{1}{2}\mathbf{x}_1 + \frac{1}{2}\mathbf{x}_2$

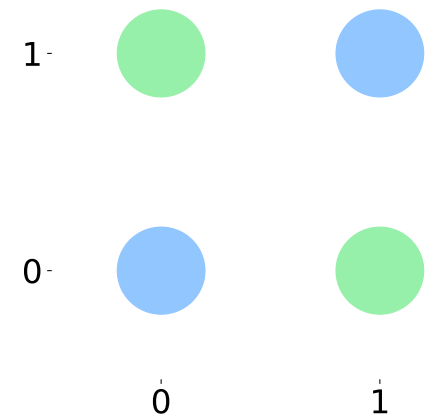
Linear models

A linear model cannot distinguish paws from background

Limitations of Linear Models

Linear models

Cannot learn XOR function

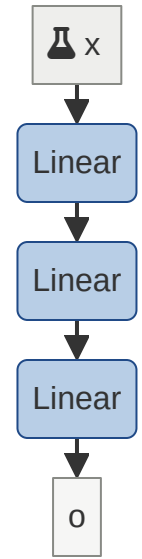


Does Adding More Linear Layers Help?

No

Combination of linear layers is still linear!

$$
\begin{aligned}
&\mathbf{W}_2(\mathbf{W}_1\mathbf{x} + \mathbf{b}_1) + \mathbf{b}_2 \\
=&(\mathbf{W}_2\mathbf{W}_1)\mathbf{x} + (\mathbf{W}_2\mathbf{b}_1 + \mathbf{b}_2) \\
=&\mathbf{W}'\mathbf{x} + \mathbf{b}'
\end{aligned}
$$



Solution: Deep Networks

Add layers that are not linear