Basic Linear Algebra

What Is Linear Algebra?

Linear algebra (in deep learning):

A mathematical language to express many operations at once.

Vector: An Array of Numbers

$$\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$$

Element-Wise Vector Operations

$$\mathbf{v} + \mathbf{w} = \begin{bmatrix} v_1 + w_1 \\ v_2 + w_2 \\ \vdots \\ v_n + w_n \end{bmatrix} \qquad \mathbf{v} - \mathbf{w} = \begin{bmatrix} v_1 - w_1 \\ v_2 - w_2 \\ \vdots \\ v_n - w_n \end{bmatrix} \qquad \mathbf{v} \cdot \mathbf{w} = \begin{bmatrix} v_1 \cdot w_1 \\ v_2 \cdot w_2 \\ \vdots \\ v_n \cdot w_n \end{bmatrix} \qquad \mathbf{v} / \mathbf{w} = \begin{bmatrix} v_1 / w_1 \\ v_2 / w_2 \\ \vdots \\ v_n / w_n \end{bmatrix} \qquad \ldots$$

Matrix: A 2D Array of Numbers

$$\mathbf{M} = \begin{bmatrix} M_{1,1} & M_{1,2} & \cdots & M_{1,m} \\ M_{2,1} & M_{2,2} & \cdots & M_{2,m} \\ \vdots & \vdots & \ddots & \vdots \\ M_{n,1} & M_{n,2} & \cdots & M_{n,m} \end{bmatrix}$$

Matrix Transpose

$$\mathbf{M} = \begin{bmatrix} M_{1,1} & M_{1,2} & M_{1,3} & M_{1,4} \\ M_{2,1} & M_{2,2} & M_{2,3} & M_{2,4} \end{bmatrix} \to \mathbf{M}^\top = \begin{bmatrix} M_{1,1} & M_{2,1} \\ M_{1,2} & M_{2,2} \\ M_{1,3} & M_{2,3} \\ M_{1,4} & M_{2,4} \end{bmatrix}$$

Row Vector: The Transpose of a Vector

$$\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} \rightarrow \mathbf{v}^\top = [v_1, v_2, \cdots, v_n]$$

Matrix Multiplication

$$C_{i,j} = \sum_{k=1}^{p} A_{i,k} \cdot B_{k,j}$$

Matrix Multiplication: Example

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \\ b_{41} & b_{42} & b_{43} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \end{bmatrix}$$

2×4 $\quad$ 4×3 $\quad$ 2×3

Vector Multiplication

A *column vector* is multiplied on the right:

$$\mathbf{M}\mathbf{v}$$

A *row vector* is multiplied on the left:

$$\mathbf{v}^\top\mathbf{M}$$

Inner product (dot product)

$$\mathbf{v}^\top \mathbf{w} = \sum_{i=1}^{n} v_i w_i$$

Outer Product

$$M = \mathbf{v}\mathbf{w}^\top$$

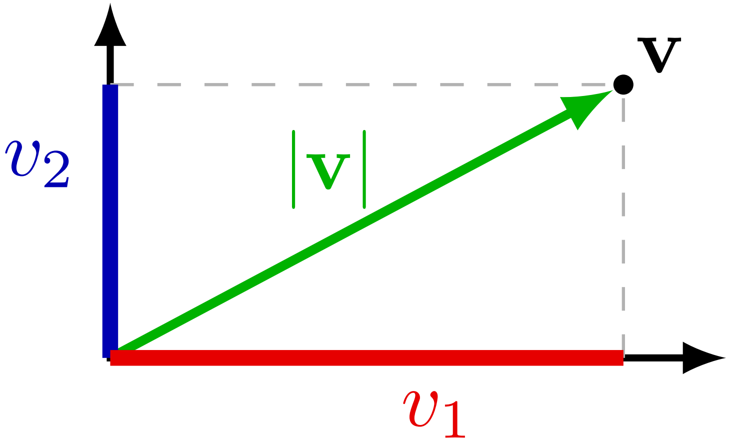
Euclidean Norm: Length of a Vector

$$\|\mathbf{v}\| = \sqrt{\mathbf{v}^\top \mathbf{v}} = \sqrt{\sum_{i=1}^{n} v_i^2}$$

Euclidean Norm: 2D Example

Length of $\mathbf{v}$:

$$\|\mathbf{v}\| = \sqrt{v_1^2 + v_2^2}$$



Frobenius Norm: Magnitude of a Matrix

$$\|\mathbf{M}\| = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{m} M_{i,j}^2}$$

Linear Algebra Through an Example

Given $a_1, a_2, \dots a_n$ compute:

$$b_i = a_i - \mu_a$$
$$\text{where} \quad \mu_a = \frac{1}{n}\sum_{i=1}^{n} a_n$$

Without linear algebra:

```
# Assume a = [a_1, a_2, ...] is given
S = 0
for v in a:
    S += v
mean = S / len(a)
b = []
for v in a:
    b.append(v - mean)
```

With linear algebra:

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b = a - a.dot(torch.ones_like(a)) / len(a)
```

With pytorch:

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b = a - a.mean()
```

Basic Linear Algebra - TL;DR

Linear algebra: a mathematical language to express many operations at once